

Magnetic Field induced Dimensional Crossover Phenomena in Cuprate Superconductors and their Implications

T. Schneider and J. M. Singer¹

Physikinstitut, Universität Zürich, Winterthurerstr. 190, CH-8057 Zürich, Switzerland

Abstract

We discuss the occurrence of crossing points in the magnetization - temperature (m, T) plane within the framework of critical phenomena. It is shown that in a two-dimensional superconducting slab of thickness d_s $m_z(\delta)$ versus temperature T curves measured in different fields $\mathbf{H} = H(0, \sin(\delta), \cos(\delta))$ will cross at the critical temperature T_c of the slab. In contrast, in a 3D anisotropic bulk superconductor the crossing point occurs in the plot $m_z(\delta)/H_z^{1/2}$ versus T . The experimental facts that 2D crossing point features have been observed in ceramics and in single crystals for \mathbf{H} close to $\mathbf{H} = H(0, 0, 1)$, but not for $\mathbf{H} = H(0, 1, 0)$, is explained in terms of an angle-dependent crossover field separating the regions where 2D or 3D thermal fluctuations dominate. The measured 2D-crossing point data are used to estimate one of the fundamental parameters of cuprate superconductors, the minimum thickness of the slab (d_s), which remains superconducting. Our estimates, based on experimental 2D-crossing point data for single crystals, reveal that this length adopts material dependent values. Therefore, experimental data for T_c and $\lambda_{\parallel}^2(T=0)$, plotted in terms of T_c versus $1/\lambda_{\parallel}^2(T=0)$ will not tend to a straight line with universal slope as the underdoped limit is approached. Implications for magnetic torque measurements are also worked out.

Key words: High- T_c cuprates, dimensional crossover, fluctuations, XY scaling

¹ Corresponding author. Tel.: +41 1 635 4017 fax: +41 1 635 5704, e-mail: jms@physik.unizh.ch

1 Introduction

It is well established that cuprate superconductors exhibit a pronounced anisotropy in their thermodynamic and transport properties. Within the framework of a Ginzburg - Landau description, this anisotropy is characterized by the effective mass anisotropy

$$M_x \approx M_y = M_{\parallel} \ll M_{\perp} = M_z, \quad \gamma = \sqrt{M_{\perp}/M_{\parallel}} \gg 1 \quad (1)$$

where $M_{\parallel,\perp}$ denotes the effective masses perpendicular and parallel to the CuO_2 layers, respectively. Transport and magnetic torque measurements (see e.g. [1] and refs. therein) revealed that γ adopts – even close to optimum doping – rather large values and exhibits a strong doping dependence, i.e. γ rises sharply by approaching the underdoped limit. Noting then that $\gamma \rightarrow \infty$ represents the 2D limit, one concludes that the materials will progressively exhibit quasi 2D properties by approaching the underdoped limit, and that at $T = 0$ an insulator to superconductor transition occurs. In this limit, the materials can then be viewed as a stack of independent superconducting slabs of thickness d_s . This feature appears to be a generic property of cuprate superconductors, and it implies a dimensional crossover from 3D-XY to essentially 2D-XY behavior as the underdoped limit is approached [1]. The scaling theory of quantum critical phenomena [2] predicts that for 2D-XY systems the relation

$$\lim_{x \rightarrow x_u} T_c(x) \lambda_{\parallel}^2(x, T = 0) = \frac{\Phi_0^2 d_s}{Q_0 16 \pi^3 k_B}, \quad (2)$$

is universal. λ is the magnetic penetration depth, Φ_0 the flux quantum, d_s the thickness of the superconducting slab (or sheet), k_B the Boltzmann constant and Q_0 adopts an universality class specific value. x is the doping variable relative to the underdoped limit x_u , corresponding to a critical endpoint where T_C vanishes. This relation appears to explain the experimentally observed trend, that for a variety of cuprate superconductors T_C plotted versus $1/\lambda_{\parallel}^2(x, T = 0)$ seems to approach a universal line as T_C decreases in the underdoped regime [3]. It was found, however, that several cuprates containing CuO chains exhibit a reduced values $1/\lambda_{\parallel}^2(x, T = 0)$ compared to the universal line [4,5]. In this context it should be recognized that the available experimental data are measured quite far away from the critical endpoint where Eq. (2) applies. Furthermore, the existence of an universal line would require a material independent value of the slab thickness d_s . Such an universal value of d_s appears to be rather unlikely due to the large variations in the chemistry within the unit cell.

We explore within the framework of critical phenomena the effect of an applied angular dependent magnetic field on the thermodynamic properties of

cuprate superconductors, with special emphasis on the angular dependence of magnetization and magnetic torque. It is shown that in a 2D superconducting slab of thickness d_s $m_z(\delta)$ versus T curves measured in different fields $\mathbf{H} = H(0, \sin(\delta), \cos(\delta))$, will cross at the transition temperature T_c of the slab, while in an anisotropic bulk (3D) superconductor the crossing point occurs in plots $m_z(\delta)/H_z^{1/2}$ versus T . The experimental fact that 2D-crossing point features have been observed in ceramics and in single crystals for \mathbf{H} close to $\mathbf{H} = H(0, 0, 1)$, but not for $\mathbf{H} = H(0, 1, 0)$, is explained in terms of an angle-dependent crossover field separating the regions where 2D or 3D thermal fluctuations dominate. The measured 2D-crossing point data are then used to estimate one of the fundamental parameters of cuprate superconductors, the minimum thickness of the slab (d_s) which remains superconducting. Our estimates, based on experimental 2D-crossing point data for single crystals reveal, that d_s adopts indeed a material dependent value. As a consequence, experimental data for T_c and $\lambda_{\parallel}^2(T=0)$, plotted in terms of T_c versus $1/\lambda_{\parallel}^2(T=0)$ should not tend to straight line with universal slope as the underdoped limit is approached. Implications for magnetic torque measurements are also worked out.

2 Angular dependence of the magnetization and crossing point phenomena

The appropriate scaling form of the singular part of the free energy density for an anisotropic bulk superconductor in an applied magnetic field $\mathbf{H} = (0, H_y, H_z)$ is [6,7]

$$f_s = \frac{Q_3^{\pm} k_B T}{\xi_a^{\pm} \xi_b^{\pm} \xi_c^{\pm}} G_3^{\pm}(z), \quad (3)$$

with the scaling variable

$$z = \frac{(\xi_a^{\pm})^2 H}{\Phi_0} \sqrt{\frac{M_a}{M_c} \sin^2(\delta) + \frac{M_a}{M_b} \cos^2(\delta)}. \quad (4)$$

$G(z)$ is an universal scaling function. We obtain for the magnetization

$$m = -\frac{\partial f_s}{\partial H}, \quad (5)$$

$$m(\delta) = -\frac{Q_3^{\pm} k_B T}{\Phi_0^{3/2}} H^{1/2} \left(\frac{M_c^2}{M_a M_b} \right)^{1/4} \left(\cos^2(\delta) + \frac{M_b}{M_c} \sin^2(\delta) \right) \frac{dG_3^{\pm}(z)}{dz} \frac{1}{\sqrt{z}}, \quad (6)$$

where we used

$$\frac{\xi_i^\pm}{\xi_j^\pm} = \sqrt{\frac{M_j}{M_i}}. \quad (7)$$

For the z -component (perpendicular to the CuO_2 -layers) the appropriate scaling form then reads

$$\frac{m_z}{H_z^{1/2}} = -\frac{Q_3^\pm k_B T}{\Phi_0^{3/2}} \left(\frac{M_c^2}{M_a M_b} \right)^{1/4} \frac{dG_3^\pm(z)}{dz} \frac{1}{\sqrt{z}}, \quad (8)$$

$$z = \frac{(\xi_a^\pm)^2 H_z}{\Phi_0} \sqrt{\frac{M_a}{M_b}}.$$

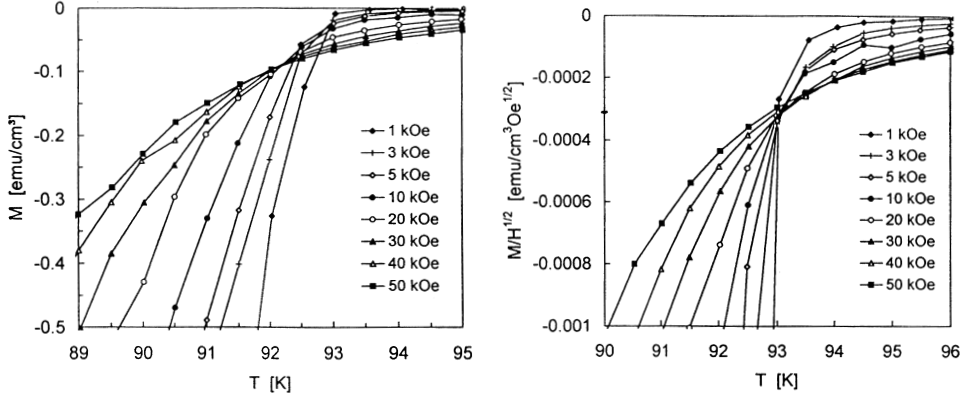


Fig. 1. m_z versus T (left panel) and $m_z/H_z^{1/2}$ versus T (right panel) in various fields, $\mathbf{H} \parallel c$, for an optimally doped $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ single crystal with $T_c = 93 \text{ K}$ (Taken from [10]).

In finite fields and close to T_c , where ξ_a^\pm diverges (i.e. $z \rightarrow \infty$), the existence of a magnetization obviously requires

$$\lim_{z \rightarrow \infty} \frac{dG_3^\pm(z)}{dz} \frac{1}{\sqrt{z}} = c_{3,\infty}^\pm, \quad (9)$$

leading to

$$\frac{m(T_c, \delta)}{H^{1/2}} = -\frac{Q_3^\pm c_{3,\infty}^\pm k_B T_c}{\Phi_0^{3/2}} \left(\frac{M_c^2}{M_a M_b} \right)^{1/4} \left(\cos^2(\delta) + \frac{M_b}{M_c} \sin^2(\delta) \right) \quad (10)$$

Thus, in a plot $m_z(\delta)/H_z^{1/2}$ versus T the data taken in different fields H will cross at T_c . This behavior has been well confirmed in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ for $\delta = 0$ [8,10]. As an example we show data from Junod et al. [10] in Fig. 1.

Using $\sqrt{M_c/M_b} \approx 8.95$, $\sqrt{M_b/M_a} \approx 0.83$ [7], resulting from magnetic torque measurements, and $m_z(T_c)/H_z^{1/2} \approx 3 \cdot 10^{-4} \text{emu/cm}^3 \text{Oe}^{1/2}$ (taken from Fig. 1) we obtain for the universal number the estimate

$$Q_3^\pm c_{3\infty}^\pm \approx 0.27, \quad (11)$$

which is close to the value 0.32 obtained in the Gaussian approximation [11].

On the other hand, considering a 2D-slab of thickness d_s , the scaling form of the singular part of the free energy density (Eq.4) reduces to

$$f_s = \frac{Q_2^\pm k_B T}{\xi_a^\pm \xi_b^\pm d_s} G_2^\pm(z). \quad (12)$$

yielding for $M_a \approx M_b = M_\parallel$

$$m_2(\delta) = -\frac{Q_2^\pm k_B T}{\Phi_0 d_s} \sqrt{\cos^2(\delta) + \frac{1}{\gamma^2} \sin^2(\delta)} \frac{\partial G_2^\pm(z)}{\partial z},$$

$$z = \frac{(\xi_a^\pm)^2 H}{\Phi_0} \sqrt{\cos^2(\delta) + \frac{1}{\gamma^2} \sin^2(\delta)}. \quad (13)$$

At the transition temperature of the slab, T_c^{slab} , ξ_a^\pm diverges, i.e. in finite fields

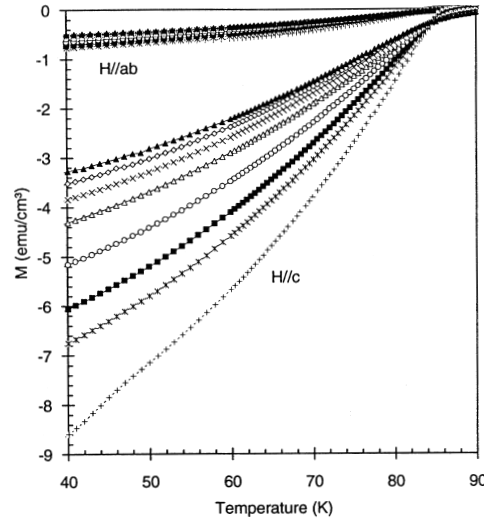


Fig. 2. m versus T for a $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ single crystal with $T_c = 85\text{K}$ in the vicinity of T_c (taken from [9]). Upper set of curves: the field is parallel to the CuO_2 planes. Lower set of curves: the field is parallel to the c -axis. $H = 1\text{kOe}$ (+) to $H = 50\text{kOe}$ (filled triangles).

$z \rightarrow \infty$. Again, the existence of a magnetization requires

$$\lim_{z \rightarrow \infty} \frac{dG_2^\pm(z)}{dz} = c_{2,\infty}^\pm, \quad (14)$$

and, thus, Eq.(13) can be rewritten as

$$m_2(\delta, T_c^{slab}) = -\frac{Q_2^\pm c_{2,\infty}^\pm k_B T_c^{slab}}{\Phi_0 d_s} \sqrt{\cos^2(\delta) + \frac{1}{\gamma^2} \sin^2(\delta)}, \quad (15)$$

where $Q_2^\pm c_{2,\infty}^\pm$ is an universal number. As a consequence, in a plot $m(\delta)$ versus T for fixed δ , the graphs taken in different fields will now cross at T_c^{slab} . To provide an estimate for $Q_2 c_{2,\infty}$, we note that the Gaussian approximation yields [12,13]

$$Q_2^\pm c_{2,\infty}^\pm = 0.52. \quad (16)$$

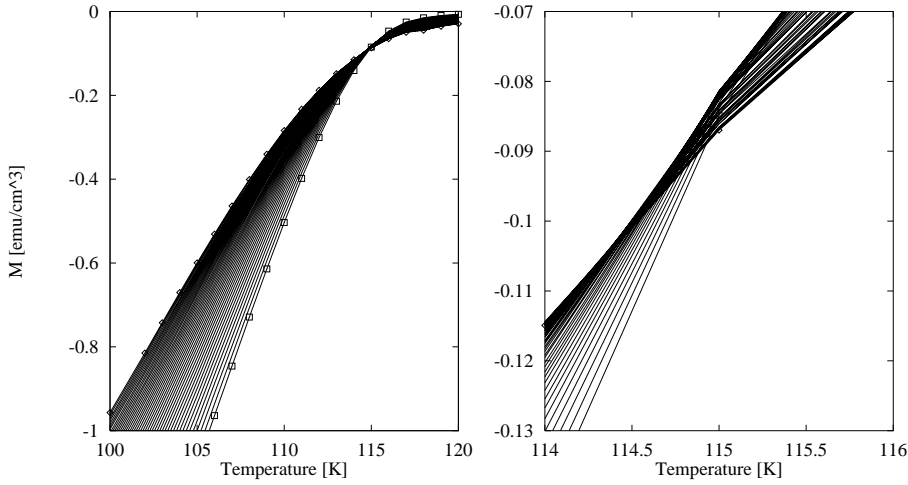


Fig. 3. Magnetization m_z vs. T for a $Tl-1223$ single crystal with $T_c = 116K$ for various applied fields $H = 10kOe$ (square) to $H = 55kOe$ (diamond) perpendicular to the CuO_2 -layers. The right panel shows a blow-up of the crossing region. We note that although $Tl-1224$ is a quite anisotropic compound, the crossing point is for lower fields rather a *region* than a point in a strict sense. Data courtesy G. Triscone [17].

In recent years considerable evidence for the occurrence of this 2D-crossing point phenomenon has been accumulated for sufficiently anisotropic materials for ceramic samples and in single crystals for fields perpendicular to the layers.

To illustrate this behavior we show in Figs. 1 - 4 experimental data for an optimally doped $YBa_2Cu_3O_{7-\delta}$ single crystal, a $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystal, a $TlBa_2Ca_2Cu_3O_9$ single crystal randomly oriented polycrystalline $HgBa_2CuO_{4+\delta}$. From Fig. 1 (left panel) it is seen that in optimally doped $YBa_2Cu_3O_{7-\delta}$, corresponding to the most isotropic material, there is no well defined 2D-crossing point, while in the more anisotropic materials, $Bi_2Sr_2CaCu_2O_{8+\delta}$, $HgBa_2CuO_{4+\delta}$ and $TlBa_2Ca_2Cu_3O_9$ for $H \parallel c$ a reasonably well defined 2D-crossing point occurs over more than three decades of the field (see e.g. Fig. 4), given the limited resolution in both, magnetization and temperature. Indeed, even in a highly anisotropic $TlBa_2Ca_2Cu_3O_9$ single crystal,

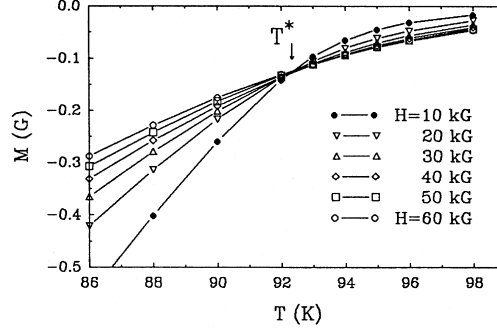


Fig. 4. \bar{m} versus T for randomly oriented polycrystalline $\text{HgBa}_2\text{CuO}_{4+\delta}$, applied field is perpendicular to the CuO_2 -layers. (Taken from [14].

the 2D-crossing point is seen to transform on a finer grid to a crossing region (Fig. 3, right panel), revealing the presence of a finite interslab coupling. Moreover, for $H \perp c$, the 2D-crossing point phenomenon is not observed, even in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$, one of the most anisotropic cuprates [9].

Before we turn to this point and the angular dependence of the magnetization in single crystals, it is useful to treat ceramic samples. Assuming randomly oriented grains the crossing point relation (15) transforms into

$$\begin{aligned} \overline{m_2}(T_c^{slab}) &= -\frac{k_B T_c^{slab}}{\Phi_0 d_s} g(\gamma) Q_2^\pm c_{2,\infty}^\pm, \\ g(\gamma) &= \frac{1}{\pi} \int_0^\pi d\delta \sqrt{\cos^2(\delta) + \frac{1}{\gamma^2} \sin^2(\delta)}. \end{aligned} \quad (17)$$

From Fig. 5 it is seen that the variations of $g(\gamma)$, in the range of the relevant γ values, is rather small.

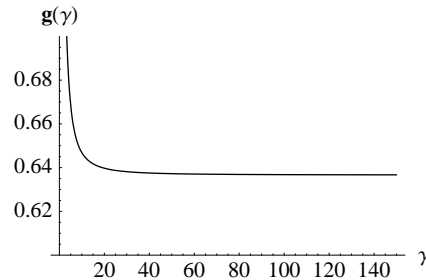


Fig. 5. $g(\gamma)$.

The observation of the 2D crossing point features in ceramic samples and single crystals with $H \parallel c$ seems to suggest a dimensional crossover, which may be understood as following: approaching the bulk transition temperature T_c

from below, ξ_c^- diverges as

$$\xi_c^-(T) = \xi_{c0}^- \left(1 - \frac{T}{T_c}\right)^{-\nu}, \nu \approx 2/3. \quad (18)$$

Lowering the temperature below T_c , $\xi_c^-(T)$ can become smaller than s , the crystallographic CuO_2 layer repeat distance. This causes a separation of the 3D bulk superconductor into nearly uncoupled 2D - superconducting slabs of thickness d_s , provided that the anisotropy γ is sufficiently large. This, in turn, renders the critical amplitude ξ_{c0}^- to be sufficiently small, so that

$$\xi_c^-(T) < s \quad (19)$$

is satisfied for $T < T_s < T_c$ ($\xi_c^-(T_s) = s$). T_s is the *slab decoupling temperature*, i.e. the order parameter fluctuations in adjacent slabs are only weakly correlated for $T < T_s$, and the system corresponds to a stack of nearly independent quasi-2D superconducting slabs of thickness d_s . In a strongly anisotropic system T_c^{slab} is close to, but lower than the bulk transition T_c . T_c^{slab} can be estimated from the bulk transition temperature in terms of the finite size expression

$$\xi_c^-(T_c^{slab}) = d_s \approx \xi_{c0}^- \left(1 - \frac{T_c^{slab}}{T_c}\right)^{-\nu}, \nu \approx 2/3. \quad (20)$$

Clearly (by definition), around T_c^{slab} two-dimensional fluctuations will dominate. Using for $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ ($T_c = 91.5 \text{ K}$) $\xi_{c0}^- = 1.64\text{\AA}$, $s = 8.2\text{\AA}$ and for $\text{HgBa}_2\text{CuO}_{4+\delta}$ ($T_c = 95.6 \text{ K}$) $\xi_{c0}^- = 0.94\text{\AA}$, $s = 9.51\text{\AA}$ [7] as well as

$$\xi_c^-(T_s) = s \approx \xi_{c0}^- \left(1 - \frac{T_s}{T_c}\right)^{-\nu}, \quad (21)$$

we find that $T_s \approx 83.3 \text{ K}$ for $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ and $T_s \approx 92.6 \text{ K}$ for $\text{HgBa}_2\text{CuO}_{4+\delta}$. A condition to observe in a bulk system a reasonably well defined 2D crossing point phenomena is then

$$T^* \approx T_c^{slab} \lesssim T_s < T_c, \quad z(T^*) \gg 1, \quad (22)$$

so that 2D critical fluctuations dominate around T^* and the 2D crossing point equations (15) and (17) apply (with $z(T^*) \rightarrow \infty$). At T_s a quasi-2D to 3D crossover occurs, and around the bulk T_c 3D-fluctuations determine the critical behavior. In agreement with experiment (Fig. 1), optimally doped $\text{YBaCu}_3\text{O}_{7-\delta}$ is not expected to exhibit 2D crossing point features, because T_s is much too far away from T_c . In $\text{HgBa}_2\text{CuO}_{4+\delta}$, however, condition (22) is partially satisfied ($T_s \lesssim T_c$), but the appearance of 2D crossing point features additionally requires $T^* \approx T_c^{slab} \lesssim T_s$, so that 2D fluctuations will dominate. $\xi_{||0}^-$ and the scaling variable z become so large that Eq. (15) or (17) applies. In $\text{HgBa}_2\text{CuO}_{4+\delta}$ the T^* phenomenon has been observed in randomly oriented

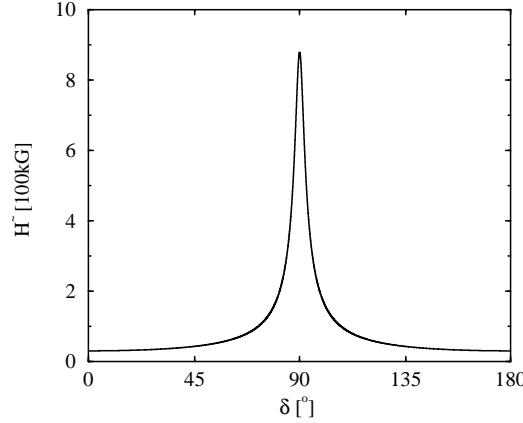


Fig. 6. \tilde{H} [T] versus δ for $\xi_{\parallel}(T^*) \approx 260.7\text{\AA}$ corresponding to $\text{HgBa}_2\text{CuO}_{4+\delta}$.

polycrystalline samples for $10kG < H < 60kG$ [14] (see Fig. 4). With the aid of Eq. (13) we introduce an angular dependent "crossover-field"

$$\frac{H^*(\delta)}{z} = \frac{\Phi_0}{\xi_{\parallel}^2(T^*)} \frac{1}{\sqrt{\cos^2(\delta) + \frac{1}{\gamma^2} \sin^2(\delta)}}, \quad (23)$$

and, noting that a dominance of 2D fluctuations at T^* requires $z \gg 1$ (Eq. (22)), we obtain the condition

$$H^*(\delta) \gg \tilde{H}(\delta) = \frac{\Phi_0}{\xi_{\parallel}^2(T^*)} \frac{1}{\sqrt{\cos^2(\delta) + \frac{1}{\gamma^2} \sin^2(\delta)}} \quad (24)$$

for the angular dependence of the field strength, where a well defined crossing point phenomenon can be observed. In other words, $\tilde{H}(\delta)$ may also be viewed as a crossover field, separating the regions where 2D- respectively 3D-fluctuations are essential. The angular dependence of \tilde{H} is shown in Fig. 6 for

$$\xi_{\parallel}(T^*) \approx \xi_{\parallel 0} (1 - T^*/T_c)^{-2/3} \approx 260.7\text{\AA},$$

with $\xi_{\parallel 0} = \gamma \xi_{\perp 0}$ and the previously used parameters for $\text{HgBa}_2\text{CuO}_{4+\delta}$.

The angular dependence of \tilde{H} , shown in Fig. 6 reveals that for moderate field strength the observation of 2D-crossing point features is restricted to ceramic samples or single crystals for fields $H_{\parallel c}$ ($\delta = 0^\circ$), while for δ values around 90° considerably higher fields are needed. Consequently, the occurrence of a reasonably well defined crossing phenomenon in ceramics or randomly oriented polycrystalline samples does not imply that 2D fluctuations dominate per se, but most likely around $\delta = 0$ only. Moreover, it should be recognized that even though 2D fluctuations may dominate around $\delta = 0$, the weak order parameter fluctuations along the c -direction render to smear out the crossing point to a

Table 1

 d_s -values for various cuprate compounds.

compound	T_c^{slab} [K]	$m_2(\delta = 0, T_c^{slab}) / T_c^{slab}$ [emu/cm ³ K]	d_s [Å]	γ	source
HgBa ₂ Ca ₂ Cu ₃ O _{8+δ}	127	$1.81 \cdot 10^{-3}$	29	52	[18]
Bi ₂ Sr ₂ CaCu ₂ O _{8+δ}	83.1	$3.54 \cdot 10^{-3}$	10	250	[19]
Bi ₂ Sr ₂ CaCu ₂ O _{8+δ}	84.5	$3.55 \cdot 10^{-3}$	10	250	[9]
Bi ₂ Sr ₂ Ca ₂ Cu ₃ O _{10+δ}	105	$1.14 \cdot 10^{-3}$	30	70	[20]
TlBa ₂ Ca ₂ Cu ₃ O _{9+δ}	115	$0.09 \cdot 10^{-3}$	44		[17]
Tl ₂ Ba ₂ CuO _{6+δ}	84.2	$3.80 \cdot 10^{-3}$	9.12	117	[21]
Bi _{1.95} Sr _{1.65} La _{0.4} CuO _{6+δ}	11.2	$2.41 \cdot 10^{-3}$	14.4		[22]

crossing region (as shown in Fig. 3), its extend will decrease with increasing γ (which, in turn, corresponds to decreasing interlayer coupling). In any case, as long as γ is finite, there will be, strictly speaking, no 2D-crossing point, because sufficiently close to the bulk T_c 3D-fluctuations dominate. In practice, however, the homogeneity of the samples and the experimental resolution are limited. For this reason the observation of reasonably well defined 2D-crossing point features in moderate fields is restricted to ceramics and single crystals with $H \parallel c$, provided that γ is sufficiently large.

Nevertheless, the observation of crossing point features offers the possibility to estimate the slab thickness with the aid of Eq. (15) or (17), provided that experimental values for $m_2(\delta = 0, T_c^{slab}) / T_c^{slab}$ or $\overline{m}_2(T_c^{slab}) / T_c^{slab}$ are available. Because the grains in ceramic samples are not necessarily randomly oriented, we listed in Table 1 some single crystal estimates. The quoted d_s -values have been derived from Eq. (15), using the Gaussian estimate for $Q_2^\pm c_{2,\infty}^\pm$ (Eq. (16)).

Noting that the quoted d_s -values depend on $Q_2^\pm c_{2,\infty}^\pm$, while their ratios are independent, the experimental data confirms the expectation according to which d_s adopts a material dependent value. This implies, combined with the universal relation (2), that experimental data for T_c and $\lambda_\parallel^2(T = 0)$, plotted in terms of T_c versus $1/\lambda_\parallel^2(T = 0)$ do not tend to straight line with universal slope as the underdoped limit is approached.

3 Magnetic torque

Next we discuss implications of two-dimensional crossing point features for the interpretation of the angular dependence of the magnetic torque. With this technique, rather accurate and extended experimental data has been ac-

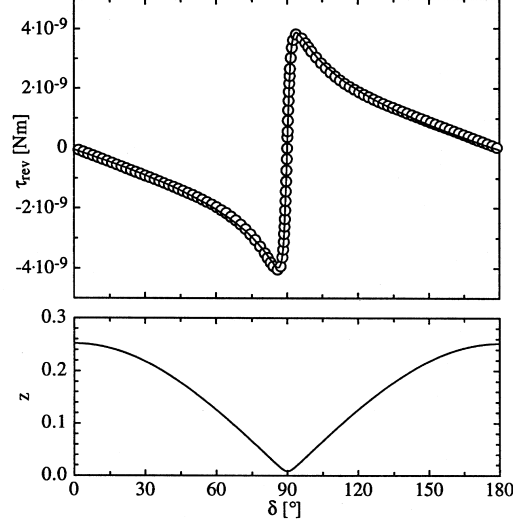


Fig. 7. a) τ_x versus δ for a $\text{HgBa}_2\text{CuO}_{4+\delta}$ single crystal with $T_c = 95.6\text{ K}$ at $H = 1.4T$ and $T = 90.9\text{ K}$. Data points have been omitted for clarity. The solid line is a fit to Eq. (25). b) z versus δ . Taken from [15].

cumulated, covering the full variation of δ .

As an example we consider $\text{HgBa}_2\text{CuO}_{4+\delta}$, where according to Fig. 6 the dominance of 2D fluctuations around $\delta = 90^\circ$ requires that $H \gg 8 \cdot 10^5 G$. Thus, for moderate field strength and $T^* \lesssim T_c$ anisotropic three-dimensional critical fluctuations are expected to dominate around $\delta = 90^\circ$. In this case the magnetic torque can be derived from singular part of the 3D free energy density, Eq. (3), yielding in the limit $z \rightarrow 0$,

$$\tau_x(\delta) = \frac{Q_3^- \Phi_0 C_{3,0}^- H}{16\pi^3 \lambda_{||0}^2} \left(1 - \frac{1}{\gamma}\right) \frac{\cos(\delta) \sin(\delta)}{\sqrt{\cos^2(\delta) + \frac{1}{\gamma^2} \sin^2(\delta)}} \ln(z). \quad (25)$$

Since the torque signal vanishes at $\delta = 0^\circ$, where the crossing point features are known to occur (see Fig. 4), this technique turns out to be rather insensitive in the regime where 2D fluctuations may dominate. To illustrate this fact we show in Fig. 7a the measured angular dependence of the magnetic torque for a $\text{HgBa}_2\text{CuO}_{4+\delta}$ single crystal with $T_c = 95.6\text{ K}$ at $H = 1.4T$ and $T = 90.9\text{ K}$. The solid line is a fit to Eq. (25), yielding $\xi_{||0}^- \approx \gamma \xi_{\perp 0}^- \approx 27\text{ \AA}$, $\gamma \approx 28.8$ (as obtained in [15]) and $Q_3^- C_{3,0}^- \approx 0.7$ for the universal number. The corresponding δ -dependence of z , shown in Fig. 7b, confirms that the data are in the appropriate range ($z \rightarrow 0$), where Eq. (25) is applicable.

The remarkable fit must be attributed to the aforementioned facts: For the field considered here ($H = 1.4T$) and for the angular regime ($\delta \approx 90^\circ$), where the magnetic torque provides a strong signal, anisotropic 3D fluctuations dominate even close to T^* . Unfortunately, in the angular regime $\delta = 0^\circ$, where around T^* 2D fluctuations are essential, the torque signal vanishes. To substantiate this

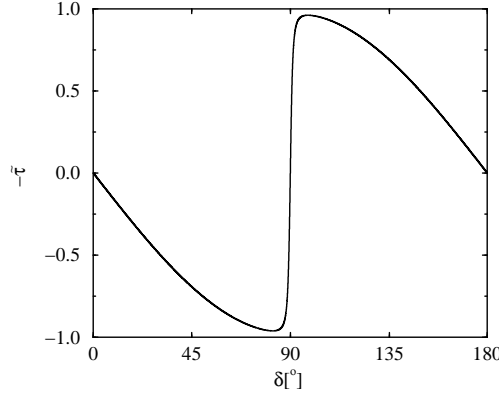


Fig. 8. $\tilde{\tau}_x$ versus δ close to T_c^{slab} for $\gamma = 50$, according to Eq.(30).

conclusion it is instructive to consider the angular dependence of the magnetic torque for a slab of thickness d_s . Close to T_c^{slab} , where Eq. (14) applies, we obtain from the scaling form of the free energy density with $m_i = -\partial f_S / \partial H_i$, $\tau_x = m_y H_z - m_z H_y$ and $M_x \approx M_y$

$$\frac{\tau_x}{H} = -\frac{k_B T Q_2^\pm c_{2,\infty}^\pm}{\Phi_0 d_s} \left(1 - \frac{1}{\gamma}\right) \frac{\sin(\delta) \cos(\delta)}{\sqrt{\cos^2(\delta) + \frac{1}{\gamma^2} \sin^2(\delta)}}. \quad (26)$$

The resulting angular dependence is shown in Fig. 8 for $\gamma = 50$. Comparing with the measured behavior in Fig. 7a, the pronounced deviations around $\delta = 90^\circ$ fully confirm our expectations that in the regime where the data of Fig. 7a were taken, i.e. for δ values where a appreciable torque signal appears, anisotropic 3D fluctuations dominate.

We are then led to the important conclusion, that around T^* the magnetic torque probes, for moderate applied fields and $\delta \approx 90^\circ$, predominantly anisotropic 3D fluctuations. To substantiate this conclusion further, we note again that even in the highly anisotropic compound, $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ the 2D-crossing point phenomenon was not observed for $H \perp c$ [9].

4 Summary and Conclusions

To summarize we have shown that an applied magnetic field,

$\mathbf{H} = H(0, \sin(\delta), \cos(\delta))$ may induce a quasi 2D to 3D crossover leading to crossing points in the magnetization - temperature (m, T) plane. In a 2D-superconducting slab of thickness d_s , the curves $m_z(\delta)$ versus T taken in different fields will cross at T_c , while in an anisotropic bulk (3D) superconductor the crossing point occurs in the plot $m_z(\delta) / H_z^{1/2}$ versus T . The experimental fact that 2D-crossing point features have been observed in ceramics and in single

crystals for \mathbf{H} close to $\mathbf{H} = H(0, 0, 1)$ but not for $\mathbf{H} = H(0, 1, 0)$ is explained in terms of a δ -dependent crossover field separating the regions where 2D respectively 3D thermal fluctuations dominate. The measured 2D-crossing point data was then used to estimate one of the fundamental parameters of cuprate superconductors, the minimum thickness of the slab (d_s) which remains superconducting. Our estimates, based on experimental 2D-crossing point data, reveal – as expected – that this length adopts a material dependent value. As a consequence, experimental data for T_c and $\lambda_{\parallel}^2(T=0)$, plotted in terms of T_c versus $1/\lambda_{\parallel}^2(T=0)$ do not tend to a straight line with universal slope as the underdoped limit is approached. Finally, we have shown that angular dependent magnetic torque measurements probe for moderate field strength and $\delta \approx 90^\circ$ predominantly anisotropic 3D fluctuations.

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